Validity of Relative Density for Quality Control of Cohesionless Soils

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Outline

• History & development of Relative Density
• How reliable is the concept?
• How reliable are the correlations?
  – Standard Penetration Test
  – Cone Penetration Test
History

• 1925: Realization of concept
  – TERZAGHI, K. (1925) *Erdbaumechanik auf Bodenphysikalisher Grundlage*, Vienna, Deuticke

• 1948: Aim
  – to bring the behaviour characteristics of soils together on a common basis in consistent and practically useful relations and to provide a tool for communications between engineers
History

• An appropriate means to define the looseness and denseness of sand or sand-gravel soils in a meaningful way because important properties were assumed to correlate quite well by this means.
History

• 1954: Formation of ASTM Committee D-18, Subcommittee 3, Section D for determining the minimum and maximum densities of sand and gravel soils

• 1964: Approval of standard by D-18

• 1969: ASTM standard for Relative Density

• 1984: Withdrawal of ASTM D2049-69
History

• 2011
  – ASTM D4253-00 (Reapproved 2006) Standard Test Methods for Maximum Index Density and Unit Weight of Soils Using a Vibratory Table.
Definition: includes three parameters

\[ D_d = \frac{e_{max} - e}{e_{max} - e_{min}} \times 100 \]

\( e_{max} = \) maximum index void ratio or the reference void ratio of a soil at the minimum index density/unit weight.

\( e_{min} = \) minimum index void ratio or the reference void ratio of a soil at the maximum index density/unit weight.

\( e = \) the in situ or stated void ratio of a soil deposit or fill.
Alternative definitions

\[ D_d = \frac{\rho_{d,\text{max}} (\rho_d - \rho_{d,\text{min}})}{\rho_d (\rho_{d,\text{max}} - \rho_{d,\text{min}})} \times 100 \]

\[ D_d = \frac{\gamma_{d,\text{max}} (\gamma_d - \gamma_{d,\text{min}})}{\gamma_d (\gamma_{d,\text{max}} - \gamma_{d,\text{min}})} \times 100 \]

\( \rho_d \) or \( \gamma_d \) = dry density/unit weight of a soil deposit or fill at the given void ratio.
ASTM Limits of Application

• Soil can contain up to 15%, by dry mass, of soil particles passing a 75-μm sieve, provided they still have *cohesionless, free-draining* characteristics.

• For determination of $\rho_{d_{\text{min}}}$, $\gamma_{d_{\text{min}}}$, $\rho_d$ or $\gamma_d$
  – 3 accepted methods are applicable to soil in which 100% of soil particles pass respectively a 75, 19 & 9.5 mm sieves.
ASTM Limits of Application

<table>
<thead>
<tr>
<th>Percent Passing</th>
<th>Grain Size (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BOULDER COBBLE</td>
<td>coarse, fine</td>
</tr>
<tr>
<td>GRAVEL</td>
<td>coarse, medium, fine</td>
</tr>
<tr>
<td>SAND</td>
<td></td>
</tr>
<tr>
<td>SILT or CLAY</td>
<td></td>
</tr>
</tbody>
</table>

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Curtin University

GFSA
ASTM words of caution

• For many types of free-draining, cohesionless soils, these test methods cause a moderate amount of degradation (particle breakdown) of the soil. When degradation occurs, typically there is an increase in the maximum index density/unit weight obtained, and comparable test results may not be obtained when different size molds are used to test a given soil.
ASTM words of caution

• The engineering properties, such as strength, compressibility, and permeability of a given soil, compacted by various methods to a given state of compactness can vary considerably. Therefore, considerable engineering judgment must be used in relating the engineering properties of soil to the state of compactness.
  
  – Note: In addition, there are published data to indicate that these test methods have a high degree of variability.
Errors

• **Systematic error** is a measure of accuracy and the difference between correct value and the measured average of a set of repeated tests.

• **Random error** is the precision of a quantity and is measured by the scatter in the results of a group of repeated tests.

• **Mistake**
Errors

SELIG, E. T. & LADD, R. S. (1973)
Yoshimi & Tohno (1973)


- $D_d$ is proportional to $(\gamma_d - \gamma_{d_{\text{min}}})/ \gamma_d \rightarrow$
  - even a small variation in $\gamma_d$ or $\gamma_{d_{\text{min}}}$ \rightarrow considerable variation in $D_d$ when $\gamma_d - \gamma_{d_{\text{min}}}$ is small; i.e. when $D_d$ is low.
  - E.g.: $\gamma_{d_{\text{min}}} = 13.5$ kN/m$^3$, $\gamma_{d_{\text{max}}} = 16.37$ kN/m$^3$, & $\gamma_d = 14.25$ kN/m$^3$, $D_d = 30\%$. If $\gamma_{d_{\text{min}}}$ is increased by 1% to $13.635$ kN/m$^3$, $D_d$ reduces to $25.8\%$ which is $14\%$ less than the initial value. In other words, the relative deviation in relative density is 14 times that of $\gamma_{d_{\text{min}}}$.
Yoshimi & Tohno: random errors

\[
\left( \frac{S_{D_d}}{D_d} \right)^2 = C_{\gamma_{d_{max}}}^2 \left( \frac{S_{\gamma_{d_{max}}}}{\gamma_{d_{max}}} \right)^2 + C_{\gamma_{d_{min}}}^2 \left( \frac{S_{\gamma_{d_{min}}}}{\gamma_{d_{min}}} \right)^2
\]

• The terms in the parentheses are coefficients of variation. \( S_{D_d}, S_{\gamma_{d_{max}}}, S_{\gamma_{d_{min}}} \) and \( S_{\gamma_d} \) are respectively the standard deviations for \( D_d, \gamma_{d_{max}}, \gamma_{d_{min}} \) and \( \gamma_d \), and \( C_{\gamma_{d_{max}}}, C_{\gamma_{d_{min}}} \) and \( C_{\gamma_d} \) are error propagation factors.

• Random errors can be reduced to any desired degree by repeating the test and averaging the results.
Yoshimi & Tohno: random errors

\[ C_{Y_{d_{\text{max}}}} = \frac{\gamma_{\text{dmin}}}{(\gamma_{d_{\text{max}}} - \gamma_{\text{dmin}})} \]

\[ C_{Y_{d_{\text{min}}}} = \frac{\gamma_{d_{\text{max}}} (1 - D_d)}{(\gamma_{d_{\text{max}}} - \gamma_{\text{dmin}}) D_d} = (C_{Y_{d_{\text{max}}}} + 1) \frac{1 - D_d}{D_d} \]

\[ C_{Y_d} = \frac{\gamma_{d_{\max}}}{(\gamma_{d_{\max}} - \gamma_{\text{dmin}}) D_d} - 1 = \left( \frac{C_{Y_{d_{\text{max}}}} + 1}{D_d} \right) - 1 \]

\[ C_{Y_{d_{\text{max}}}} = 4.7 \text{ (typical clean sand)} \]
Yoshimi & Tohno: systematic errors

\[
\frac{\Delta D_d}{D_d} \bigg|_{\Delta \gamma_{d_{\text{min}}} = \Delta \gamma = 0} = \frac{\gamma_{d_{\text{min}}}}{\gamma_{d_{\text{max}}} - \gamma_{d_{\text{min}}} + \Delta \gamma_{d_{\text{max}}}} \frac{\Delta \gamma_{d_{\text{max}}}}{\gamma_{d_{\text{max}}}}
\]

\[
\frac{\Delta D_d}{D_d} \bigg|_{\Delta \gamma_{d_{\text{max}}} = \Delta \gamma = 0} = \frac{(1 - D_d) \gamma_{d_{\text{max}}}}{D_d (\gamma_{d_{\text{max}}} - \gamma_{d_{\text{min}}} - \Delta \gamma_{d_{\text{min}}})} \frac{\Delta \gamma_{d_{\text{min}}}}{\gamma_{d_{\text{min}}}}
\]

\[
\frac{\Delta D_d}{D_d} \bigg|_{\Delta \gamma_{d_{\text{max}}} = \Delta \gamma_{d_{\text{min}}} = 0} = \frac{\gamma_d \gamma_{d_{\text{min}}}}{(\gamma_d - \gamma_{d_{\text{min}}}) + (\gamma_d + \Delta \gamma_d)} \frac{\Delta \gamma_d}{\gamma_d}
\]

\[\Delta D_d / D_d = \text{relative deviation}\]
Yoshimi & Tohno: systematic errors

Influence of systematic errors in limiting densities on relative density
Tavenas et al. (1973)

Due to the very large variability of the relative density between laboratories, the comparison of relative densities measured by different laboratories were totally non-significant. There were important practical implications of this fact: all established correlations between relative density and various properties of cohesionless soils such as standard penetration index, point resistance in a static penetration test, friction angle, modulus of compressibility, shear wave velocity, etc., are useless to anyone but the operator who has established them, since that person is the only one who can reproduce the relative density of the considered soil with sufficient accuracy.
It appeared that due not so much to the variability of the minimum and maximum unit weights but essentially to the formulation of relative density itself, the resulting accuracy of this parameter was so poor that its use was related to major uncertainties (the best case was of ideal material such as the tested fine sand, and was deemed to be practically meaningless in most of the other cases.

Tavenas et al. (1973)

Practical problems of relative density

• Difficult & costly to implement at depth
• Difficult & costly to implement below groundwater level
• Costly to repeat sufficient number of times to reduce random errors

→ Correlations: a statistical relation between two or more variables such that systematic changes in the value of one variable are accompanied by systematic changes in the other. Correlations are not physical laws or theorems, they are simply statistical relations and only meaningful once their scatters, deviations and variances are known.
Soil properties are not based only on $D_d$

- Fines content
- Grain size & shape
- Grading & grading curve shape
- Effective vertical or horizontal stresses
- Mineralogy
- Compressibility & crushability
- Cementation
- Over consolidation
- Age
Grain shape

\[ e_{\text{max}} \text{ & } e_{\text{min}} \text{ as a function of grain with } C_u=1.4 \]

Grain shape


**Effect of particle shape on** $e_{\text{max}}$ **and** $e_{\text{min}}$ **from gradational and particle shape characteristics**
Grain shape & gradation


$e_{\text{max}}$ and $e_{\text{min}}$ from gradational and particle shape characteristics
Particle size

Fines content


Correlation between limiting dry unit weights and fines content
SPT & relative density

How reliable are samples extracted from the SPT split spoon?


$$D_d = \sqrt{\frac{N}{17 + 0.25\sigma'\nu}}$$
Different correlations yield very different results

Lacroix & Horn (1973)
How good is the SPT for determining $D_d$?

Holtz (1973): “First, I think that everyone should recognize that the Standard Spoon Penetration Test is a relatively crude test and no one should expect to determine the relative density of sands to the nearest one percent or anything like that. When Mr. Gibbs and I developed a set of correlations to take into account the effect of overburden pressures, we never indicated that the sets of curves developed at that time were necessarily applicable to all cohesionless soils under all conditions. Second, we always stressed the relative density trends indicated by the Standard Penetration Test (SPT) values rather than the specific individual values. Third, I wanted to point out that Mr. Gibbs and I are not particular "promoters" of the SPT, although we think it is useful for certain types of foundation investigations, and at certain stages of investigation.”

Skempton (1986)

\[ D_d = \sqrt{\frac{N}{\alpha + b\sigma'_{v}}} \]

<table>
<thead>
<tr>
<th>Sand</th>
<th>Tested</th>
<th>( D_{50}: ) mm</th>
<th>UC*</th>
<th>Fines: %</th>
<th>( D_t: )</th>
<th>( N_1 )</th>
<th>( \frac{N_1}{D_t^2} )</th>
<th>( \frac{N}{D_t^2} )</th>
<th>( ER_t )</th>
<th>( \frac{(N_1)_{60}}{D_t^2} )</th>
<th>( \frac{(N_2)_{60}}{D_t^2} )</th>
<th>( \frac{N_{60}}{D_t^2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>PR</td>
<td>Wet</td>
<td>2.0</td>
<td>5.3</td>
<td>0</td>
<td>0.4</td>
<td>7.5</td>
<td>47</td>
<td>30 + 22( \sigma'_{v} )</td>
<td>1.1†</td>
<td>8</td>
<td>52</td>
<td>33 + 24( \sigma'_{v} )</td>
</tr>
<tr>
<td>GHC</td>
<td>Dry and moist</td>
<td>1.5</td>
<td>5.5</td>
<td>0</td>
<td>0.4</td>
<td>6.5</td>
<td>40</td>
<td>18 + 22( \sigma'_{v} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SCS</td>
<td>Wet</td>
<td>0.51</td>
<td>2.5</td>
<td>4</td>
<td>0.4</td>
<td>7</td>
<td>44</td>
<td>21 + 24( \sigma'_{v} )</td>
<td>1.1†</td>
<td>7.5</td>
<td>48</td>
<td>23 + 26( \sigma'_{v} )</td>
</tr>
<tr>
<td>RBM</td>
<td>Wet</td>
<td>0.23</td>
<td>1.8</td>
<td>2</td>
<td>0.4</td>
<td>5.5</td>
<td>34</td>
<td>16 + 17( \sigma'_{v} )</td>
<td>1.1†</td>
<td>6</td>
<td>37</td>
<td>17 + 19( \sigma'_{v} )</td>
</tr>
<tr>
<td>GhF</td>
<td>Dry</td>
<td>0.3</td>
<td>7</td>
<td>14</td>
<td>0.4</td>
<td>4.5</td>
<td>28</td>
<td>15 + 18( \sigma'_{v} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Uniformity coefficient.
† Includes a correction for no liners.

Grain size, age & consolidation

• At a given relative density and overburden pressure, $N$ values are higher for sands with larger grain sizes ($D_{50}$)

• Ageing of sand will increase the SPT blow counts

• Over Consolidation

\[
D_a = \sqrt{\frac{N}{\alpha + C_{oc} b \sigma'_v}}
\]

\[
C_{oc} = \frac{1 + 2K_0}{1 + 2K_{ONC}} \quad K_{ONC} = 1 - \sin \phi' \quad K_0 = K_{ONC}(OCR)^{\sin \phi'}
\]

Tokimatsu & Yoshimi (1983): fines content

\[
D_d = 0.21 \sqrt{\frac{N}{0.7 + \frac{\sigma'_v}{98}} + \frac{\Delta N_f}{1.7}}
\]

<table>
<thead>
<tr>
<th>Fines content (%)</th>
<th>(\Delta N_f)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-5</td>
<td>0</td>
</tr>
<tr>
<td>5-10</td>
<td>interpolate</td>
</tr>
<tr>
<td>10-</td>
<td>0.1F_c + 4</td>
</tr>
</tbody>
</table>

- Tokimatsu and Yoshimi themselves have not demonstrated confidence in their proposed equation and note that its application is yet to be proven.

Scatter of data


What is Schmertmann (1976)?


who has actually read the paper
Comparison of correlations


<table>
<thead>
<tr>
<th>$\sigma ' \nu$ (kPa)</th>
<th>$q_c$ Villet-Mitchell /Schmertmann (1978)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>20%</td>
</tr>
<tr>
<td>100</td>
<td>1.7</td>
</tr>
<tr>
<td>200</td>
<td>2.0</td>
</tr>
<tr>
<td>300</td>
<td>2.4</td>
</tr>
</tbody>
</table>
Most famous correlation: Baldi et al. (1986)

\[ D_d = \frac{1}{C_2} \ln \left[ \frac{q_c}{C_0 \sigma'} \right] \]

- Tests carried out
  - in calibration chambers
  - on Ticino and Hukksund sands
  - on normally consolidated and over consolidated sands

\( \sigma' = \) effective vertical stress if the sand is normally consolidated or as the effective horizontal stress or effective mean stress if the soil is over consolidated

Most famous correlation: Baldi et al. (1986)

Normally consolidated Ticino sand

\[
D_d = \frac{1}{2.41} \ln \left( \frac{q_c}{157\sigma'_v^{0.55}} \right)
\]

Normally consolidated Hukksund sand

\[
D_d = \frac{1}{3.29} \ln \left( \frac{q_c}{86\sigma'_v^{0.53}} \right)
\]
Jamiolkowski et al. (2001)

- Ticino sand, Hukksund sand and Toyoura sand

\[
D_d = \frac{1}{3.10} \ln \left[ \frac{q_c/98.1}{17.68 \left( \sigma'_v/98.1 \right)^{0.50}} \right]
\]

Jamiolkowski & Pasqualini: “a quality control program based only on the evaluation of relative density can be inadequate; a better estimation of the densification of sands is possible if the effects of the stress and strain history induced on the improved soil by compaction are considered.”


Effect of carbonate sand

Comparison of $q_c$ of calcareous QS and silica TS at equal $D_d$

Conclusion: Do not rely on relative density as a criterion

- Due to its formulation relative density is prone to large errors
- The relationships between relative density and field tests are not unique and are strongly influenced by other parameters such as:
  - fines content
  - grain size & grain shape
  - grading & grading curve shape
  - effective vertical or horizontal stresses
  - mineralogy
  - compressibility & crushability
  - cementation, over consolidation & age
Thank You